WIMIN 2019 Abstracts

Plenary Speakers

Erica Graham, Bryn Mawr College

Building and breaking the ovulatory cycle: modeling insights

> The ovulatory cycle is the result of a tightly regulated system of crosstalk between reproductive hormones generated in the brain and ovary. The complex hormone feedback gives rise to the characteristic oscillatory behavior of the menstrual cycle. Altered hormone regulation may disrupt the cycle, leading to abnormal ovulation and/or fertility. The sheer complexity of this system poses a challenge to identifying precise mechanisms of dysfunction. However, mathematical approaches have been useful in examining pathophysiology, even when precise biological mechanisms are unknown. I will discuss mathematical models of ovulation in the context of general dynamics, the nature of dysfunction, clinical implications, and an algorithmic approach for data- and biology-driven model reduction.

Allison Henrich, Seattle University

It's all fun and games until someone becomes a mathematician

> As former MAA President Francis Su recently reminded us, PLAY is essential for human flourishing. Whether you are a poet or a scientist, a grandparent or a child, play can powerfully enrich your life. For mathematicians, play is essential for building intuition. For undergraduates, play can inspire a desire to get involved in mathematical research. The world of knots provides fertile ground for understanding these connections. Playing games on knot diagrams can give us intuition about knotty structures, while learning about the theory of knots can reveal the "magic" behind rope tricks and excite us to learn more.

Student Speakers

Lydia Ahlstrom, Keene State College

A Mathematical Investigation of Sol LeWitt's Wall Drawing 413

When one can find a mathematical concept in a work of art, it will enrich and illuminate both the mathematics and the art. Sol LeWitt was a 20th century artist whose work is closely associated with Minimalism and Conceptual art. His Wall Drawing 413, which can be viewed at the Massachusetts Museum of Contemporary Art in North Adams, MA, consists of 64 squares each of which is divided into four smaller squares. Only four colors are used to fill the small squares. This talk will begin with an overview of Lewitt's art, followed by how Wall Drawing 413 can be interpreted in terms of permutations and groups.

Caira Anderson, Smith College

Modeling Producer-Scrounger Behavior

Kathryn Beck & Lisa Cenek, Dickinson College & Amherst College

Chorded Pancyclic Properties in Claw-Free Graphs

A graph G is (doubly) chorded pancyclic if G contains a (doubly) chorded cycle of every possible length m for $4 \leq m \leq |V(G)|$. In 2018, Cream, Gould, and Larsen completely characterized the pairs of forbidden subgraphs that guarantee chorded pancyclicity in 2-connected graphs. In our work, we show that the same pairs also imply doubly chorded pancyclicity. We further characterize conditions for the stronger property of doubly chorded (k, m)-pancyclicity where every set of k vertices in G is contained in a doubly chorded m-cycle for all $4 \leq m \leq |V(G)|$. In particular, we examine forbidden pairs and degree sum conditions that guarantee this recently defined cycle property. This work was completed as a part of the 2019 Lafayette College REU and supported by the NSF under grant number 1560222.

Talia Blum, MIT

Unlikely Intersections and Portraits of Dynamical Semigroups

Classical algebraic dynamics studies the behaviors of single rational functions under iteration. Of particular interest are the possible phase portraits of preperiodic points; Baker proved that with finitely many exceptions every preperiodic portrait can be realized by a complex rational function. Following recent work by Hindes and Hyde-Zieve, we investigate portraits for several rational functions simultaneously acting on the same point set. In this case, we no longer expect these portraits to be realizable for arbitrary rational functions. This leads to several questions, such as which portraits with several rational functions have realizations, and what properties do the spaces of all such realizations possess? We used a computer cluster to determine realization spaces for all portraits with several points and two polynomials of low degree. Surprisingly, many portraits had realization spaces with higher than expected dimension. We will present three main results: a sequence of portraits with positive-dimensional realization spaces for multiple rational functions acting on arbitrarily many points; a classification theorem for the realizable dimension of two-image portraits; and a realizable portrait with 28 quadratic polynomials acting on four points.

Tori Day, UMass Amherst

Modular Deformation Problems

Given a newform f and a choice of a prime p, Deligne and Serre constructed a semisimple two dimensional residual (mod p) Galois representation associated to f. It is profitable to study lifts of this representation to $GL_2(A)$ for certain rings A, which can be done using the deformation theory of Galois representations. This theory has played a central role in the proofs of both the Taniyama-Shimura Conjecture and Serre?s Conjecture. In this talk, we will discuss the interplay between congruences among newforms and certain modular deformation problems.

Olivia Del Guercio, Smith College

An Invariant of p-colorable Knots

In this work we compute a knot invariant known as the dihedral linking number for all 3-colorable knots up to 12 crossings. Generally, a linking number is used to describe how many times two distinct knots wind around each other. When a knot has a valid 3-coloring, it lifts to two knots in a separate three-dimensional space known as a branched cover. The dihedral linking number is the linking number of these two knots. This number could potentially allow for the differentiation of mutant knots. We are also looking into the dihedral linking numbers produced by coloring knots with any prime number of colors, which will allow for analysis of a greater number of knots.

Patrick Dragon, UMASS

What percentage of triangles are acute?

Suppose we generate a triangle at random. What is the probability it will be acute? Polling humans is an unreliable method for generation of random triangles, but is still worth investigation. We conjecture that human-generated triangles will over-represent the isosceles, right, and equilateral cases. In this talk, we will explore several approaches to the question, including some classic geometric approaches and computer simulations.

Tayler Fernandes Nunez, Northeastern University

Towards a Database of Belyi Maps

A Belyi map is a rational function with at most three critical values; we may assume these are zero, one, and infinity. A Dessin d'Enfant is a planar bipartite graph on the sphere obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from zero to one. Such graphs can be drawn on the sphere by composing with stereographic projection. This project sought to either create or expand on a database of such Belyi pairs, their corresponding Dessins d'Enfant, and their monodromy groups. We did so for up to degree N = 5 in the hopes of generating an algorithm to generate Dessins from monodromy triples.

Julie Fucarino, Wellesley College

An Agent-Based Model of Pollen Competition in Arabidopsis thaliana

In 2016, Swanson et al. showed that when an Arabidopsis thaliana stigma is pollinated with equal amounts of pollen by two accessions, Columbia and Landsberg, Columbia pollen sire disproportionately more seeds. This phenomenon is known as nonrandom mating. Previous experiments have investigated nonrandom mating by examining how pollen performance traits such as proportion of pollen germinated, time to germination, and pollen tube growth rates differ between these two accessions. In addition, bioenergetics, such as the energy supplied to pollen tubes from the pistil during fertilization, likely also magnify competition. While plant fertilization is well-studied, the exact mechanics of pollen competition remain unknown. Using an agent-based model, we aim to identify the traits that cause pollen from one accession to sire more offspring than pollen from another accession and to what extent these traits contribute to this process. We calibrate our model against a number of parameters from empirical data to observe the output of seed siring proportions from mixed pollinations; we compare these values to those found in the literature. Our model can also be extended to predict seed siring proportions for other accessions of Arabidopsis thaliana given data on their pollen performance traits.

Sarah Glidden, Smith College

Automated Identification of Landslides

Landslides occur worldwide and pose threats to property, transportation, and people. Currently, geologists must manually comb through digital elevation models to identify landslides. This process is time-consuming and prone to error. Our project lays the foundation for the creation and implementation of an automated landslide identification tool. We use elevation data collected from three counties in Virginia to create logistic models that classify a first-order stream as a landslide or non-landslide. This presentation focuses on the identification of landslides through the quantification of roughness through elevation, gradient, Laplacian, and eigenvalues. I will then discuss the limitations of our work and the next steps for this project.

Scott Greenhalgh, Siena College

A class of novel differential equation compartmental models of infectious disease transmission

Anna Grim, Brown University

Convergent Message Passing Algorithms for Probabilistic Inference in Graphical Models

Nitzan Hirshberg, University of Maryland, College Park

Approximating the Many Body Problem Through Refining Couple Cluster Theory

Here, we present a proof of principle through prototype calculations. CC calculations scale as N7, and are therefore become expensive very quickly. The only computationally affordable systems have about 30 particles. This is unhelpful for quantum chemists, nuclear physicists, and condensed matter physicists who are interested in large systems that contain a large number of atoms. In general, quantum chemists use CC in the frequency or energy space. We hope that our approximate real-time method will improve upon the efficiency of the standard CC methods, while retaining accuracy. We have confirmed that the new method of calculations produces that exact same result as the analytic solution for the 3-state/2-electron model. Through working with the 4-state/2-electron model, we have confirmed that the equations are correct, but we must be careful with which v_{pq}^{rs} we keep because of issues that arise when the valence-valence interactions are zeroed out. An ancillary goal is to expand the ability of the cumulant expansion which works very well in larger systems but produces inaccurate results in smaller systems. By observing the regimes where the cumulant expansion and CC are both accurate, we may be able to better approximate the cumulant expansion in smaller systems.

Freda Li, Wesleyan University

Quadratic and Hermitian Forms

Alexandra Newlon, Colgate University

The Fibonacci Quilt Game

GaYee Park, UMass Amherst

q-analog of factorization problems

We can count the number of ordered factorizations of an n-cycle in the symmetric group into transpositions. The result is especially interesting because it is equivalent to the number of trees with n labeled vertices. We will observe the relation of trees and factorizations and also observe a similar problem in q-analog, GL(n,q).

Jessica Rattray, Vassar College

Research in Knot Theory: Unknotting and Region Crossing Change

Take a string and tie a knot. If you connect the loose ends together, you have a mathematical knot in 3-space. The tabulation of knots is based on how many crossings they have. The simplest version of a knot is called the unknot, a closed loop with zero crossings. How do you unknot a knot that has no loose ends? We focus on achieving the unknot from a knot diagram through an operation called region crossing change. This operation allows us to change crossings in specific regions for any knot diagram. How do we calculate how a knot is unknotted? We define a new invariant called the multi-region index that measures how complex a knot is with respect to the region crossing change operation. The multi-region index of a knot is the fewest number of crossings in a set of regions that must be changed in order to achieve the unknot. Our main theorem provides a computable lower bound for the multi-region index to help us know when it is minimal. We also show that the multi-region index of a knot is at most twice the crossing number of the knot.

Garcia Sun, Smith College

Matchings and Springer fibers

Andrew Tawfeek, Amherst College

Developments on Characterizing Equivalent Discrete Morse Functions on Graphs

Discrete Morse theory is the study of analog of Morse Theory developed for CW complexes by Robin Forman in 2002. In this project, we restrict our attention to 0- and 1-cells, i.e. graphs, and further develop the subject by classifying discrete Morse functions, the main object of study. Then we not only re-prove fundamental results of the subject with basic Graph Theory (instead of homology groups) such as the Weak Morse Inequalities; but present new results in the subject on counting the number of equivalence classes of discrete Morse functions, relating the count to the graph Laplacian and automorphism groups. The method of counting the number of classes can be generalized to discrete Morse functions on simiplicial complexes, which could then lead to counting classes on CW complexes.

Asya Vitko, Wesleyan University

Counting Distinguishing Colorings of Cycles

Dawit Wachelo, Amherst College

Graph Quantum Mechanics and Von Neumann Entropy

Danielle Wiley, Keene State College

HS Sequences

A sequence of nonnegative integers begins with 2 followed by 3. The product is 6 and the sequence now becomes 2, 3, 6. The next product $3 \times 6 = 18$ creates two values 1 and 8 and the sequence is extended to 2, 3, 6, 1, 8. The next product $6 \times 1 = 6$ results in 2, 3, 6, 1, 8, 6. The sequence will continue infinitely in this manner because each multiplication produces a single or double-digit nonnegative integer, so either one or two values will always be added on the "tail" of the sequence. This sequence first appeared in a problem devised by Hugo Steinhaus. A sequence generated in this way can begin with any two nonnegative single digits and it will be referred to as an HS sequence. This talk will analyze the long-term behavior of these sequences.