Math for Grades 1 to 5 Should Be Art

Jim Henle

The Mathematical Intelligencer

ISSN 0343-6993
Volume 42
Number 4

Math Intelligencer (2020) 42:64-69
DOI 10.1007/s00283-020-09980-2

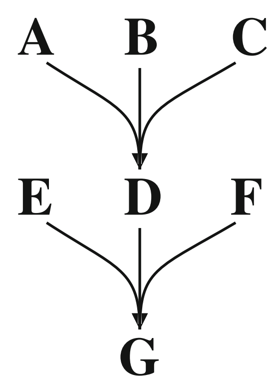
Math for Grades 1 to 5 Should Be Art

Jim Henle

This is a column about the mathematical structures that give us pleasure. Usefulness is irrelevant. Significance, depth, even truth are optional. If something appears in this column, it's because it's intriguing, or lovely, or just fun. Moreover, it is so intended.

I have a wild idea.[1] I think the focus of mathematics education before grade 6 should be mathematical art. Students should see mathematical art, play with mathematical gems, explore mathematical treasures, and they should themselves create mathematical art. They could be taught stuff too, but that would be incidental. Most of their time should be spent with intriguing, entertaining, enticing mathematical structures. And definitely: no sheets of arithmetic problems.

That's my proposal. My argument for it is logically tight:



If that doesn't convince you, I have details. Here are the first premises:

A:	B:	C:
If you love math, you'll do math.	If you do math, you'll get SMART.	SMART is better than knowing.

A isn't controversial. No defense is needed here.

B is generally believed to be true, though I have seen one dissent. There are multiple studies supporting it, but they're not convincing. Each study has its own conception of "smart" and often a limited conception of "math."

What I mean by SMART is problem-solving intelligence—the confidence to tackle problems, the keenness to test reasonable and unreasonable ideas, the comfort with logical

➢ **Jim Henle**, Department of Mathematics and Statistics, Burton Hall, Smith College, Northampton, MA 01063, USA.
e-mail: pleasingmath@gmail.com

[1]Not my first.

https://doi.org/10.1007/s00283-020-09980-2

structures, and the self-confidence to doubt one's solutions. It's hard to measure all that.

I take **B** on faith. I hope you will too. Some additional clarification is coming.

C is special. It has only recently become true. It used to be that knowing stuff was the essence of being educated, but today, knowledge is cheap. It's everywhere on the web. And intelligence is at a premium.

Knowledge may get you your first job. But that job will disappear in five years. To keep up with new approaches, new perspectives, and new paradigms, you will need SMARTS.

I'll say much more about **C** later, but I think you can see where **A**, **B**, and **C** are going.

> **D:**
> The primary goal of any math class should be for the students to fall in love with mathematics.

The *primary* goal. Teaching stuff every now and then is okay. Students in love with math will demand it. Teachers shouldn't get in the way of passionate students. But the primary goal is genuinely primary. If not love, at least you want students to actively enjoy mathematical ideas.

For the conclusion, I join **D** with two additional premises:

> **E:**
> Math art is perfect for fostering math love.

> **F:**
> Even young kids can create beautiful math art.

E is almost tautologically true. By (our) definition, mathematical art is the creation of mathematical structures that bring pleasure, that excite and intrigue. That's love.

Of course, not all beautiful works of math art will appeal to young kids. But there are so many games, so many puzzles, so many cool ideas to attract them. They don't have to like them all. It's art, after all. Kids have taste. But every child's mind can be filled with wonder.

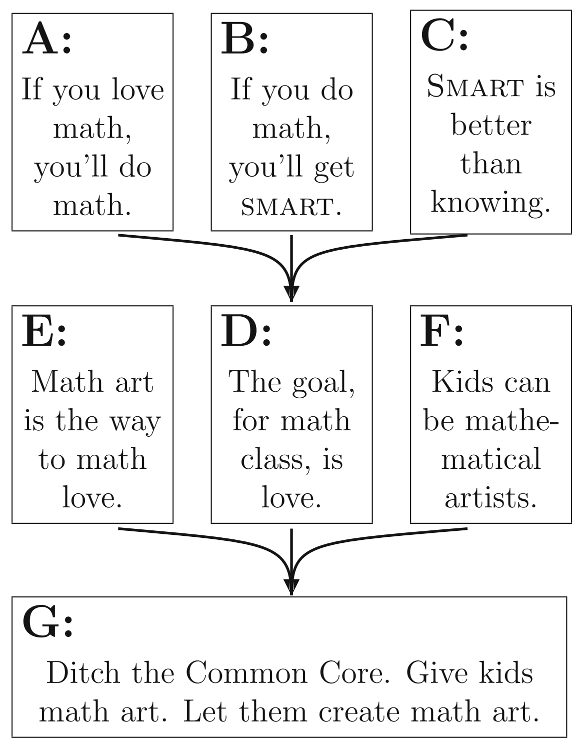
F may be a surprise to the reader. A work of math art is a mathematical structure. We define a mathematical structure as anything that can be described completely and unambiguously. That's challenging. Can kids do it?

Yes. Yes they can.

But before I go further, let me admit a personal lack of expertise. Despite what I said earlier, knowing a few things is sometimes important. I did teach grades 5 through 8 for a few years in my youth. But my ignorance in the area of primary education is immense.

Recently, though, I worked with what may be a more challenging set of students: college juniors and seniors who long ago decided they had neither ability nor affection for mathematics. But in one semester of math art, the students of that description created mathematical structures, ART, that simultaneously engaged Smith College math majors and sixth-graders at the Smith campus school.

I have art projects for kids, for prospective math artists. I'll get to them in a moment, but here is the full argument:



I still owe you:

Additional clarification for **B**.

A more detailed argument for **C**.

A convincing collection of projects for **F**.

After that, I have:

A few more thoughts on **G**.

B: (Do Math and You Get SMART)

B can be confused with "Take math and you'll earn more money" or "If you do well in math then you're smart" or "If you're smart you'll be good in math" and sometimes "If you believe you're smart then you'll be good in math." These are quite different. I'm asserting only that if you stretch your mind working on logical problems, you'll get smarter at problem-solving.

My point is that problem-solving in mathematics is not a special math-specific skill. There are, as George Pólya so brilliantly charted, important math-specific techniques.[2] But the essentials of problem-solving in mathematics are shared by all disciplines. I make this argument in detail elsewhere.[3]

C (SMARTS over Knowledge)

The history of math pedagogy has a cyclical quality. Liberalization has been conceived and promoted over and over. It has always met determined opposition. The idea recurs that math class should give students pleasure, but so does the idea that pain in mathematics is unavoidable. Ultimately, liberal curricula foundered on the admitted fact that students did need skills, that work demanded skills, and that life favors those who know math and can calculate.

But the situation has changed, and the change has come quite recently. It's not knowledge anymore that society demands, it's intelligence, it's SMARTness. The robotic revolution looming over us is pushing this hard. What is clear today is that the workers of tomorrow will need keen problem-solving skills. Revolutions in manufacturing, transport, energy, entertainment, medicine, media, force us to learn and relearn constantly.

When calculators first entered classrooms, many educators and much of the public worried that computational skills would start to disappear. They were right. But few realized that in the future that wouldn't matter. Today, the web is never far away. Mathematical assistance is freely available and is not limited to calculation. Anyone can summon Wolfram Alpha to solve complex equations and answer difficult questions.

Many wonder whether today's students are as prepared as students were twenty or thirty years ago. The truth is that they are *better* prepared. Their computational and algebraic skills are not as good, but they're better problem-solvers. Give 40-year-old college professors apps they've never seen before and they will be paralyzed. But a first-year college student, given an unknown app, will start hitting buttons, clicking on boxes, and watching what happens. She'll soon get it working.

I've had students use a pocket calculator to multiply 7 times 8. I can still teach them calculus.

Employers definitely look for SMARTS. Math majors at Smith get jobs easily and often in areas that have nothing to do with what they actually know. One of our graduates applied to a consulting group in some nontechnical field. She was asked, "Can you give us a rough idea of how many gas stations there are in the U.S.?"

The question was really a test of the student's problem-solving strength. She thought for a few minutes. In her mind, she counted the number of stations in her hometown, then used the population of the town and the population of the U.S. to come up with a rough approximation. They hired her.

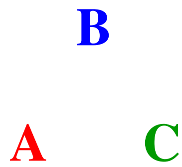
I know a software firm that recruits new math PhDs who have no coding skills. They're hired because they're SMART. I know of a brilliant number theorist who had her pick of academic positions when she left her postdoc at MIT. She chose instead to join a hedge fund that recruited her for her brilliance.

F (Kids Can Create Math Art)

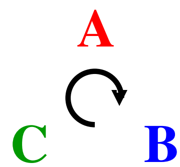
You don't have to explain "mathematical structure" to children. Here are meaningful, engaging projects that embody the idea of mathematical structure intrinsically.

1. Most kids know the game tic-tac-toe. Let them play it until they get tired of it. Then suggest that they invent a new game, something like tic-tac-toe, but different. Suggest that they invent a game that is better, more interesting than tic-tac-toe. They can do this. And when they do, they create a mathematical structure. If the rules to their games aren't clear, you can point this out. They'll understand, and clarity is the essence of mathematical structure.
2. There are lots of simple games that can serve in place of tic-tac-toe, among them dots and boxes, take-away games, crossing games. I know many. *You* know many.
3. Give the students a paper envelope. Tell them to carefully take it apart to see how it once was just a piece of paper. Then suggest that they invent their own envelope, their own way of folding and gluing paper to form something capable of holding letters. Having done that, ask them if they can invent a process that will produce an envelope of any given size.
4. A similar adventure can be had from examining a box and designing new boxes.
5. Give the students a box and paper in which to wrap it. Then ask them to find another way of wrapping the box. And then another way. Maybe they can find a way that uses less paper. Their method might involve cutting the paper into pieces. A nice challenge is to find a way that uses minimal paper but paper in a single piece.
6. In general, paper-folding, paper-cutting, and origami offer many possibilities for mathematical creativity.
7. An earlier column mentioned that Zoltan Dienes led children into realms of modern algebra with dance.[4] His simplest dance just involved three kids.

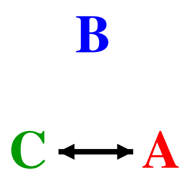
[2] George Pólya, *How to Solve It*, Princeton University Press, 1945
[3] *The Proof and the Pudding*, Princeton University Press, 2015
[4] "A Mathematical Art," *Mathematical Intelligencer* 41:3 (2019), 28–32.



There were two dance moves. One was a clockwise rotation,



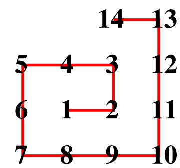
the other was to switch the dancers occupying two specific positions:



Invite your kids to explore the dance, let them combine dance steps, and essentially *play*. As they do, they will be living inside the nonabelian group of order 6 (don't tell them).

When they have exhausted their explorations of Dienes's dance, invite them to invent their own dances and explore them.

8. Dance possibilities will increase when a ribbon or a rope is added—or hats that can be passed around ...
9. Kids can invent puzzles. Do they like sudoku? Or kenken? They can invent their own variation. They will understand and appreciate the aesthetic of the unique solution.
10. Logic puzzles are fun and there are magazines and books devoted to them. Ambitious kids can try their hand at inventing their own.
11. Puzzle-invention is one of the most active areas of mathematical art today—see my earlier columns on this.[5]
12. There is a sort of catch-all genre of mathematical art I call "doodles." Doodles are simply messing around with numbers, or diagrams, or structures. A famous example is Stanisław Ulam's number doodle, in which he just began writing the natural numbers in a rectangular spiral,



and was then surprised to see many prime numbers lining up on diagonals.

Another example is a "see and say" sequence. My favorite starts out with a random number or set of numbers:

31

Then you say what you see, "one three and one one." You write that down:

1311

Now you say this new set (three ones and one three) and write it down.

3113

And you keep going. Amazingly, at some point you will wind up repeating, that is, you will end up at a sequence that is its own description.[6] In this case, it's

21322314

Kids can explore this. They can invent their own doodles.

13. Geometric doodles can be great fun. Any attempt to fill the plane with figures, polygons, etc. generates a doodle.
14. Bouncing rays inside a rectangle is a doodlish idea. Bouncing inside an ell-shaped region adds fun. Throw in some mirrors.
15. And then there is numeration. Kids can invent a method of naming numbers.

A friend of mine, Tom Weiner, used to tell his sixth-graders every year the tale of the mysterious mathematician Professor Étoile, who invented a new way of notating numbers using the letters A, B, C, D, and O. His method was lost, Tom explains, when he fell off a boat and was eaten by an octopus.

Tom then divided the class into groups. Each group was tasked with creating its own system of numeration using A, B, C, D, and O. After a week, they presented their methods in a mini-math conference. I was usually invited as the keynote speaker.

Tom did this every year with great success. His students came up with many creative systems.

[5] "Puzzle Ninja Ninja," *Mathematical Intelligencer* 40:3 (2018), 63–67, and "A Flowering of Mathematical Art," to appear in the *Mathematical Intelligencer* 42(1): 36{40, 2020.

[6] This sequence is a variant of one that J. H. Conway studied. In Conway's sequence, you move from left to right, so that 1 3 1 1 is described as "one one, one three, and two ones" or 1 1 1 3 2 1. Conway's sequences don't settle down but grow longer and longer.

16. My earlier column on numeration methods offers more ideas that students can use (and that I used when teaching middle school).[7]

More on **G** (Primarily Art for Primary School)

I have argued for spending most of math time in grades 1–5 with mathematical art, enjoying it, exploring it, and creating it. I don't expect this proposal to make much headway. Many parents remember their own math classes as a time of strife. They may feel that the trials and the successes and the failures have made them what they are today. They may feel that their children won't succeed without a similarly painful experience.

It was once thought that school, unaccompanied by physical pain, would not result in learning. It was a partial advance in education when physical pain was replaced by psychological and spiritual pain. The curriculum I'm recommending would be, I think, a more exciting and affirming advance.

A serious problem in implementing my program is generating teachers for it. In addition to knowing a variety of structures and avenues for exploration, teachers must have the self-confidence to play with totally new ideas. They must be willing to let students lead them. Even if my curriculum is adopted, it will take a generation to develop such teachers.

It's essential to give exploring students as much freedom as possible. They should decide what it is they like. They should investigate the structures that call to them. Show them many lovely bits and let them choose which to explore. The goal, after all, is for them to fall in love. It's *romance*. The standard curriculum isn't romance. It's an arranged marriage.

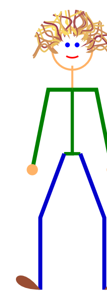
Let me emphasize that my recommendation is not to "make math fun." I am not dressing up the standard curriculum. I am not beautifying anything that is unlovely or presented in an unlovely way. I'm really discarding the standard topics. I'm substituting art for arithmetic and computation.

The specific recommendation of grades 1–5 is probably naive. You and anyone else are welcome to play with it. I was inspired by the 1929 experiment of Louis Bénézet.

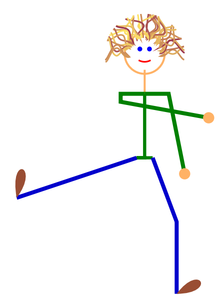
Bénézet was the superintendent of schools for Manchester, New Hampshire. He persuaded the principals of several elementary schools to halt all instruction in arithmetic for grades 1 through 5. He urged replacing it with free discussion—what he called "oral composition"—together with an occasional real-world word problem, what we call today "quantitative reasoning."

The experiment was, by his account, a huge success. Students were given their arithmetic in sixth grade and by the end of that year were significantly better at mathematics than those who had passed through the (then) standard curriculum.[8]

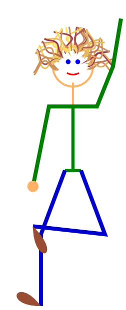
Finally, I have a little something for those who have come this far. My keynote address at Tom Weiner's class math conference involved presenting a numeration system of my own. I stand in front of the class



and ask for someone to give me a number. Maybe a student says "nineteen!" I immediately change position.



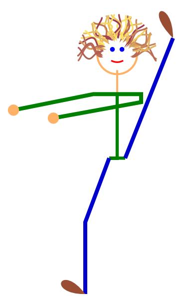
I ask for another number. I hear "Seventy-two!" I change again.

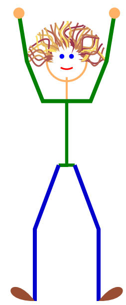


In quick succession come "One hundred and thirty-three!" and "Ten!"

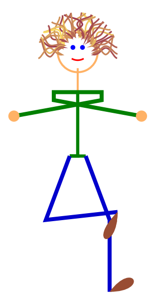
[7]"Numeralogy," *Mathematical Intelligencer* 41:4 (2019), 22–27.

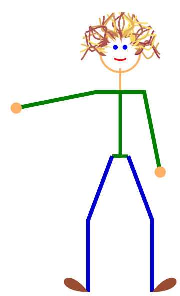
[8]See http://www.inference.org.uk/sanjoy/benezet/three.pdf.





Then, “Fifty-five!” and “One!”

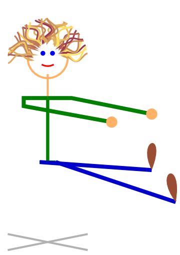




Then someone calls out “A thousand!” at which point I say, “Sorry! I can’t go that high!”

They ask how high I can go. I tell them 255.

They want to see 255.



I can only hold this for a split second. It takes a little practice.

Have you figured out the system? I’ll post an explanation on the column website, www.math.smith.edu/~jhenle/pleasingmath.

I would love to hear from readers what they think of the proposal. I expect disagreement, corrections, perhaps even ridicule. Possibly guarded approval. But all comments sent to me at pleasingmath@gmail.com are most welcome.

Jim Henle
Smith College
Northampton, MA
USA
e-mail: jhenle@smith.edu

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.