

A Flowering of Mathematical Art

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A Flowering of Mathematical Art

JIM HENLE , AND CRAIG KASPER

This is a column about the mathematical structures that give us pleasure. Usefulness is irrelevant. Significance, depth, even truth are optional. If something appears in this column, it's because it's intriguing, or lovely, or just fun. Moreover, it is so intended.

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I¹ have argued that the creation of mathematical structures is an art. The previous column discussed a tiny genre of that art: numeration systems. You can't describe that genre as "flowering." But activity is most definitely blossoming in another genre. Around the world, hundreds of artists are right now creating puzzles of subtlety, depth, and charm. We are in the midst of a renaissance of logic puzzles.

A Renaissance

The flowering began with the discovery in 2004 in England, of the discovery in 1980 in Japan, of the invention in 1979 in the United States, of the puzzle type known today as sudoku. By that time, the Japanese puzzle magazine *Nikoli* had already birthed a colony of puzzle artists. That story is described in an earlier column on Alex Bellos's *Puzzle Ninja*.² The popularity of sudoku and similar puzzles sparked an explosion of puzzle construction and puzzle invention.

Puzzles of a logical nature have a long history, arguably thousands of years.³ The more recent history of logical puzzles includes those of Lewis Carroll and Raymond Smullyan, as well as the "logic grid puzzles" of the last century.

The puzzle phenomenon we celebrate in this column is about puzzles—

- that are logical, that is, they can be solved purely by logical deduction (e.g., The filomino puzzle in Figure 1),⁴
- that are concrete, that is, they involve spatial relationships visualized geometrically (e.g., The dogleg puzzle in Figure 2),⁵
- and while they may involve letters, they are independent of language (e.g., The monogram puzzle in Figure 3).

The first two characteristics are significant in the attraction of these puzzles. The third is critical to the global nature of the puzzle-solving and puzzle-constructing community, and was especially important to the first Japanese puzzlists.

As puzzles of this sort started to catch on globally, the World Puzzle Championships (and later, the World Sudoku

¹JH.

²Puzzle Ninja Ninja. *Mathematical Intelligencer* 40:3 (2018).

³For example, Epimenides's liar paradox ("This statement is false") and the counterdilemma of Euathlus can be considered logic puzzles, albeit with no obvious answers.

⁴The rules for this and the other puzzles will come a little later in this column.

⁵All the puzzles here were constructed by the second author (CK). With the exception of one, all were composed for this column. The exception is the Dogleg, one of CK's contributions to the 2018 United States Puzzle Championship contest.

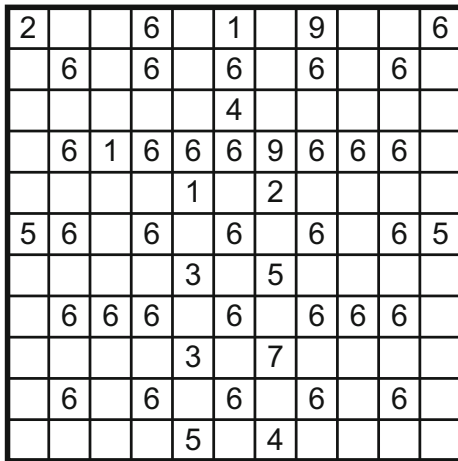


Figure 1. Filomino puzzle.

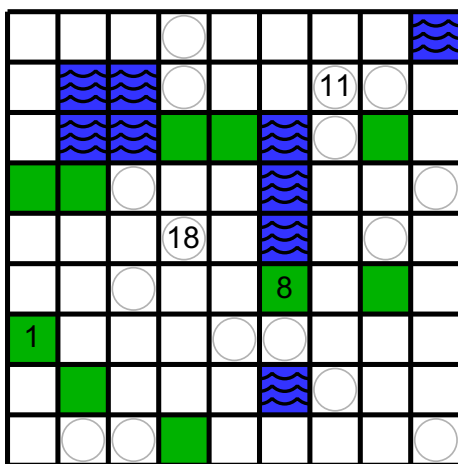


Figure 2. Dogleg puzzle.

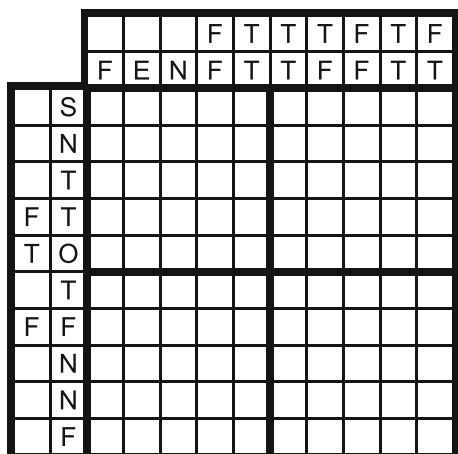


Figure 3. Monogram puzzle.

Championships) were founded, along with the World Puzzle Federation (which serves in a capacity similar to that of the International Olympic Committee for these competitions). These competitions brought solvers and constructors together, broadening relationships and transmitting and cross-pollinating the inventiveness of the puzzle constructors as they did so.

The Public

The market for puzzles today is huge, but as with other arts, segmented. There is, first of all, a mass market for puzzle types such as sudoku, kakuro, and kenken. Thousands of new puzzles appear every day in newspapers, magazines, books, and on the web. That these works of art are unabashedly consumerist is no doubt part of their popular appeal: think Warhol, not Wyeth.

Unsurprisingly, some of the ongoing appeal of the mass-market puzzle to the ordinary puzzle consumer lies in the comfortable consistency of the puzzles thus consumed. Not everyone solves puzzles to be challenged; some do so to relax, and for those solvers, relaxing puzzles are the order of the day.

But some solvers engage with puzzles to be challenged, to learn, and to discover. These solvers crave puzzles that are not only fun, but novel and interesting. This may mean closely inspecting the shelves at their local Barnes and Noble in search of puzzle collections that match their taste for the exotic, or it may mean scouring the internet for puzzle blogs and other sources of artisanal puzzles.⁶

It is an open secret that many, if not most, mass-market puzzles in the most popular categories are produced by software. The software that creates these puzzles is usually written to produce puzzles that are consistently within a certain defined standard, especially with respect to difficulty. If the software is well written, a puzzle constructed by computer will bear no identifying marks, but over time, the sameness of the puzzles will eventually become evident. More sophisticated puzzle solvers will notice and appreciate the artistic touch and personality that comes through in human-constructed puzzles. Accordingly, as solvers grow in experience and sophistication, their appetite for novel and artistic puzzles continues to grow. And as the appetite for these puzzles is growing, so too are the ranks of puzzle makers learning how to construct these puzzles and then share their constructions with the world.

The Art

Though it may not always seem that way, puzzle construction can be expressive, although unlike more conventional art forms, some wide swaths of emotion are relatively inexpressible, even in more explicitly verbal types of puzzle such as crossword puzzles. (More often, however, puzzle construction expresses ideas, and this is

⁶While the thrust of this article is the emerging artistry of the growing ranks of logic puzzle constructors, we nevertheless refer to their puzzles as artisanal to emphasize the deliberate handmade craftsmanship of the works they produce.

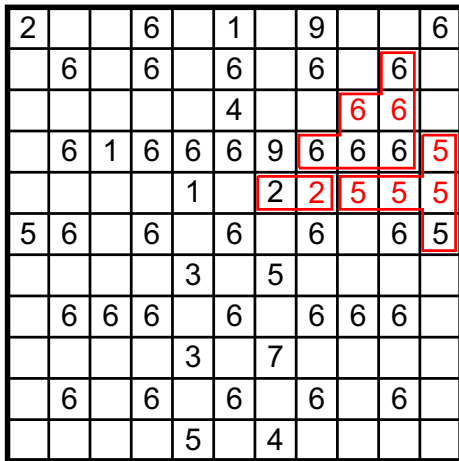


Figure 4. Possible, but ultimately false, start to a filomino puzzle.

likely where it is of the most interest to the mathematical mind.)⁷

Despite limitations, it is possible to express emotion even in the abstract style of puzzle we are discussing here. For instance, a puzzle may be like a joke, in which there is a build-up to a sudden, unexpected, and mischievous resolution.⁸ More often, however, a puzzle may be like a story that the hero overcomes, pacing that goes neither too fast nor too slow, unexpected plot twists, and eventual triumph. A great puzzle constructed in this way may elicit the emotions associated with a vacation or an adventure—pleasure, mystery, excitement, or simply a sense of calm.

The first puzzle presented here is a filomino puzzle. In a filomino puzzle, the grid is to be filled completely with numbers so that the shape formed by each set of horizontally and vertically connected identical numbers is a polyomino (a connected set of squares) comprising that number of squares. Figure 4 offers a possible start that will lead, however, to a violation of the last rule: polyominoes that share a side must have different areas. You can see that this attempt will fail, because the six polyomino will share a side with any polyomino containing the 6 in the upper right-hand corner.

Our expectation is that you will definitely feel the puzzle composer's hand in this one as you solve it: there is a motif that is definitely visible in the puzzle itself, as well as a definite flow to much of the solving, and a devious surprise or two along the way.

The dogleg puzzle is an abstraction of an 18-hole golf course. The tees are the green squares, the circles are the holes, and the blue squares are water hazards. A solution must start at the first tee, the 1, and draw a dogleg path (a path making exactly one right-angled turn) to a hole labeled 1, then to a neighboring green square labeled 2, and so on. For this you will have to provide additional

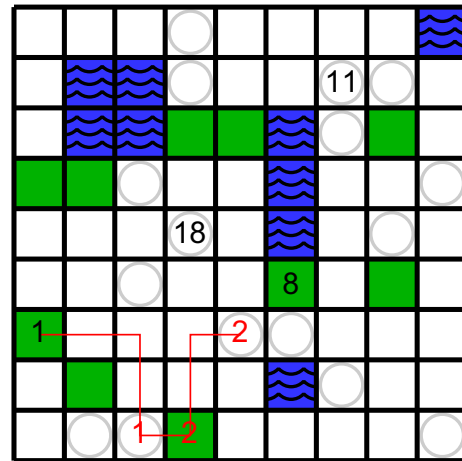


Figure 5. Dogleg puzzle.

labels. You will also need to add greens and holes (there are currently not enough). Figure 5 is a possible start.

The paths cannot cross and must eventually cover all the squares except the water hazards.

The third puzzle presented here is a (solvable) joke of sorts. It looks like a nonogram puzzle, in which sets of numbers above the columns and beside the rows enumerate all sequences of connected shaded squares in the finished puzzle to form a picture. But those are letters, not numbers—what could they stand for? The footnote at the end of this sentence explains everything; it is the reader's choice whether to read the footnote or attempt a solution without it.⁹

One will often find that the best puzzles combine elements of all the above—an amusing motif, some interesting ideas, and a story of sorts, told by the solver, who navigates through long and winding paths of narrow or dependent deductions deliberately set up by the constructor for this purpose. (In this sense, solving the puzzle can be seen as appreciating the art of the puzzle, even if the solver is not deliberately setting out to do so.)

Similarly, one could argue that there is a strong parallel between puzzle composition and music composition. The puzzle solver is akin to the performer of a musical piece. There are paths to follow and yet there is agency. The solver feels the presence of the constructor in the same way that the musician communes with the composer. The difference with puzzles, of course, is that the performer and the audience are the same.

The Artists

Without the artists, of course, the artisanal puzzles would not be forthcoming. These artists tend to toil in relative obscurity (relative to those in Japan, at least), not entirely unlike the painters of the Renaissance—though even those painters

⁷Philosopher of art Dana Leibsohn, when I (JH) asked her in conversation whether modern artists, in contrast to nineteenth-century artists, had focused on truth rather than beauty, responded, "Not truth; ideas."

⁸This is most often true of so-called hunt-style puzzles, in which so-called aha! moments of insight are a staple of the genre, but it can also be the case elsewhere.

⁹CK: This puzzle is a nonogram puzzle in which the first letter of the number is given instead of the number itself. In other words, it's a *monogram*.

often received patronage or commissions, which are rather uncommon in twenty-first-century North America.

Unlike painting, however, abstract logic puzzle construction has a relatively low barrier to entry; the 10,000 hours so famously posited by Malcolm Gladwell as being necessary to master a skill are not required. Accordingly, apprenticeship or mentorship arrangements are relatively uncommon for logic puzzle constructors (as opposed to crossword puzzle construction, where mentorship is much more commonplace); logic puzzle constructors are generally self-taught. This seems to have resulted in many if not most constructors outside of Japan having either a math background or a computer science background; both fields involve exploration and understanding of systems, which are often combined with a learn-by-doing approach. It also results in many of these constructors learning their craft, and experiencing their most productive period as constructors, during the prime learning period between late high school and post-secondary graduation.

How, then, do you find the artists toiling in obscurity? Fortunately, not all of them are toiling in obscurity. The American teams for the World Puzzle and Sudoku Championships select one or more members every year for the team through the U.S. Puzzle Championship and U.S. Sudoku Championship, located online at <http://wpc.puzzles.com>.

While the U.S. Sudoku Championship puzzles recently have all been the work of one constructor (Wei-Hwa Huang), the U.S. Puzzle Championship typically includes puzzles from between eight and twelve different constructors. USPC puzzles are often particularly novel and/or reflect the distinctive styles of their constructors and are worth a look from the casual solver for that reason alone. USPC puzzles can also be quite challenging to complete, as befits a championship, so be prepared for that if you try them.¹⁰

You can also go where the solvers go. A number of prominent high-quality constructors have found their way to Grandmaster Puzzles,¹¹ a fledging puzzle publisher that produces puzzle e-books and has a puzzle blog that operates on a partially free/partially subscription basis. It is very much worth the puzzle aficionado's time. You may also be interested in capsule biographies of its founder, puzzle champion Thomas Snyder,¹² and its regular contributors.¹³

Both the USPC and Grandmaster puzzles will give you access to polished abstract logic puzzles. There are also a few other sources of polished puzzles worth mentioning here: the Griddle¹⁴ also includes word puzzles and the challenging Adalogical Aenigmas series from Pavel's Puzzles.¹⁵ Both of these are patron-supported through Patreon.

The sites that we have listed feature relatively established constructors offering their creations; many of the constructors whose work you will see there have been constructing puzzles for five or even ten years or more. If you want to see new work by new or relatively unknown constructors, you need to be able to find where they post their puzzles. As with most lower-volume self-publishing, this usually takes the form of blogging. Often, the best way to find one of these blogs is through another blog that links to it. Meanderlawn's blog¹⁶ has a substantial list of them in its right sidebar, conveniently listed with the most recently updated blog first.

As you can see, there are a lot of high quality handmade puzzles being produced, by a lot of people. But does an archetype of the puzzle artist exist? Even though the people who make the puzzles share a number of characteristics—they are curious, intellectually active, creative people—the answer, in our experience, is “not really.” But just as with other arts, this is a good thing: it is the unique experiences and perspectives of the artists that make the art (and the puzzles) unique.

One More

Our third rule for puzzles was that they should be free of language, although the “monogram” puzzle violates this injunction in a tiny way (for example, one needs to know that the English words for 6 and 7 begin with an s. So in Figure 6 we'll give you one more puzzle, a “wordoku.” It uses letters, but letters bear no more meaning in a wordoku than numbers do in a sudoku.

						A		E
I			G			R		L
	N			E		T	I	C
		T			N			
R			E			C		
G				L			E	
	E				L			R
						I		
C	R	A	I	G				

Figure 6. A wordoku puzzle.

¹⁰The top solver in this year's contest solved 16 of the 20 puzzles in the few hours of the competition. The first author of this column took a month and a half to match that.

¹¹Online at gmpuzzles.com.

¹²See gmpuzzles.com/blog/about-thomas-snyder.

¹³See gmpuzzles.com/blog/about-our-contributing-puzzlemasters.

¹⁴See <https://thegriddle.net>.

¹⁵Available at <http://pavelpuzzles.com/aenigmas>.

¹⁶At meanderlawn.blogspot.com.

(The R, G, and E on the left are there just to guarantee that the puzzle's answer is unique.)

Answers to all the puzzles can be found on the column website, math.smith.edu/~jhenle/pleasingmath/.

Questions and comments are most welcome at pleasingmath@gmail.com.

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